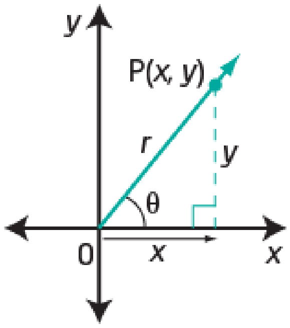
Trigonometric Ratios



Suppose θ is any angle in standard position, and P (x,y) is any point on its terminal arm, at a distance of r from the origin. The value of r can then be determined using the Pythagorean Theorem, $r = \sqrt{x^2 + y^2}$

PRIMARY TRIGONOMETRIC RATIOS

The three primary trigonometric ratios can be defined in terms of x, y, and r as follows:

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} \qquad \cos\theta = \frac{adj}{hyp} = \frac{x}{r} \qquad \tan\theta = \frac{opp}{adj} = \frac{y}{x}$$

RECIPROCAL TRIGONOMETRIC RATIOS

The three reciprocal trigonometric ratios can be defined in terms of x, y, and r as follows:

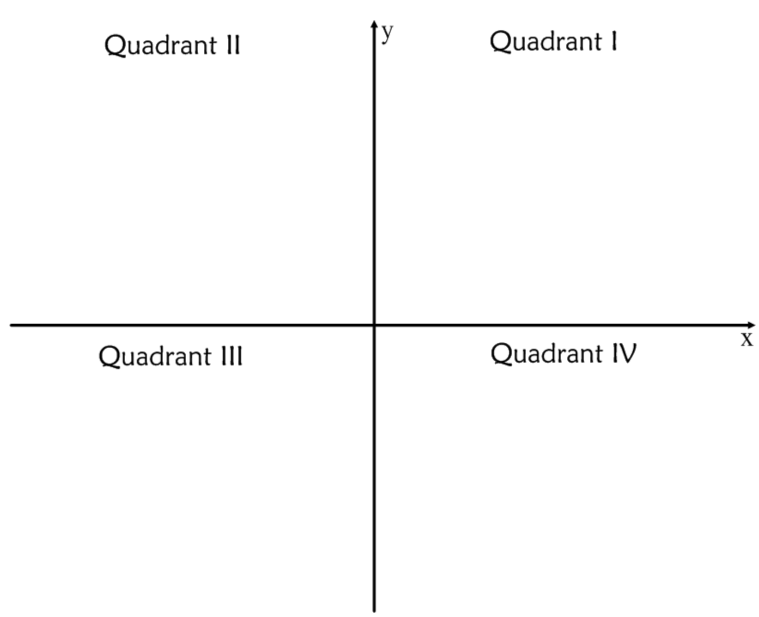
$$\text{cosecant } \theta = \frac{1}{\sin\theta} \qquad \text{secant } \theta = \frac{1}{\cos\theta} \qquad \text{cotangent } \theta = \frac{1}{\tan\theta}$$

OR, abbreviated and in terms of x, y, and r:

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

THE CAST RULE

The six trigonometric ratios of any angle in the *first* quadrant are always positive, however, this is not the case in the other quadrants. For each quadrant, we will determine the sign for each of the *primary* trigonometric ratios and summarize the results with the CAST rule.



We can determine the six trigonometric ratios for any angle in standard position using:

i. the coordinates of the point where terminal arm intersects the unit circle, and/or

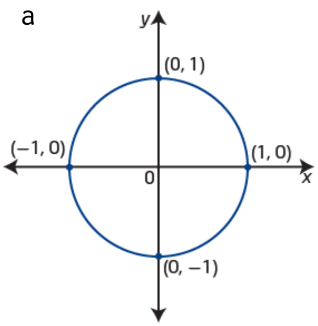
ii. the special triangles

Example 1: Determine the Trigonometric Ratios for Angles in the Unit Circle

The point $A\left(\frac{-12}{13}, \frac{5}{13}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle θ in standard position.

a. Draw a diagram to model the situation.
b. Determine the values of the six trigonometric ratios for θ. Express your answers in lowest terms.

Solution:

a. a 	b.

Example 2: Exact Values for Trigonometric Ratios

Exact values for the trigonometric ratios can be determined by using the unit circle or special triangles.

Determine the exact value for each trigonometric ratio.

a. $\sin\frac{5\pi}{6}$	b. $\cos\frac{-2\pi}{3}$
c. $\csc 315°$	d. $\tan 180°$

Example 3: Approximate Values of Trigonometric Ratios

You can determine approximate values for sine, cosine and tangent using a calculator. Remember to set your calculator to either degree or radian setting, depending on the question. To find the value of a trigonometric ratio for cosecant, secant or cotangent, use the appropriate reciprocal relationship.

For example, $\csc 4.1 = \dfrac{1}{\sin 4.1} = -1.2220$ (Set calculator to radians)

Determine the approximate value for each trigonometric ratio. Give your answers to four decimal places.

a. $\tan \dfrac{9\pi}{5}$	b. $\sin 220°$	c. $\cos 1.25$	d. $\sec(-110°)$

Example 4: Evaluating Trigonometric Ratios

Determine the *exact* value for each of the following trigonometric expressions.

a. $\sin 45^{o} \cos 45^{o} + \sin 30^{o} \sin 60^{o}$

b. $\dfrac{2\sin^2 \frac{3\pi}{4} + \cos^2 \frac{5\pi}{6}}{\cos \frac{2\pi}{3}}$

c. $\dfrac{3\cos 180^{o} + \sin 135^{o}}{\sin 30^{o}}$

Example 5: Calculating Trigonometric Values for Points Not on the Unit Circle

The point A(6, –8) lies on the terminal arm of an angle θ in standard position.

a. What is the exact value of each trigonometric ratio for θ?

b. Determine θ in the domain $-4\pi \le \theta \le 4\pi$.

Solution:

Example 6: Find Angles Given Their Trigonometric Ratios

Determine the measures of all angles that satisfy the following.

a. $\cos\theta = 0.598472$ in the domain $0 \le \theta < 2\pi$. Give your answers to the nearest tenth of a radian.

b. $\sin\theta = -0.819152$ in the domain $0° \le \theta < 360°$. Give your answers to the nearest degree.

c. $\cos\theta = \frac{-\sqrt{2}}{2}$ in the domain $0 \le \theta < 4\pi$. Give exact answers.

d. $\tan\theta = \frac{1}{\sqrt{3}}$ in the domain $-180° \le \theta < 180°$. Give exact answers.

e. $\csc\theta = -\frac{2}{\sqrt{3}}$ in the domain $-2\pi \le \theta < \pi$. Give exact answers.

Solution:

a. $\cos\theta = 0.598472$ in the domain $0 \le \theta < 2\pi$. Give your answers to the nearest tenth of a radian.

b. $\sin\theta = -0.819152$ in the domain $0° \le \theta < 360°$. Give your answers to the nearest degree.

c. $\cos\theta = \frac{-\sqrt{2}}{2}$ in the domain $0 \le \theta < 4\pi$. Give exact answers.

d. $\tan\theta = \frac{1}{\sqrt{3}}$ in the domain $-180° \le \theta < 180°$. Give exact answers.

e. $\csc\theta = -\frac{2}{\sqrt{3}}$ in the domain $-2\pi \le \theta < \pi$. Give exact answers.